GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

Mark Mark	Description
M1	Method mark (awarded for showing correct process)
A1	Accuracy mark (awarded for correct answer dependent on M mark)
B1	Independent mark (awarded for correct answer/statement regardless of method)

Mark Scheme & Worked Solutions

Q	Working / Guidance	Mark	Notes
1	$a = 3, b = 8, c = -5$	**M1**	Correct substitution into quadratic formula.
	$x = \frac{-8\pm\sqrt{8^2-4(3)(-5)}}{2(3)} = \frac{-8\pm\sqrt{64+60}}{6} = \frac{-8\pm\sqrt{}}{6}$	**M1**	Correct evaluation of discriminant ($\sqrt{124}$ or $2\sqrt{31}$).
	$x = 0.5225...$ or $x = -3.189...$	**A1**	Both answers correct to 2 d.p.
	$x = 0.52$ **or** $x = -3.19$		

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

2	$\frac{3(x+4)-2(2x-1)}{(2x-1)(x+4)}$ $= \frac{3x+12-4x+2}{(2x-1)(x+4)}$ $= \frac{14-x}{(2x-1)(x+4)}$ *(or* $\frac{-x+14}{2x^2+7x-4}$*)*	**M1** **M1** **A1**	Common denominator with numerators adjusted. Correct expansion of brackets (watch $-2(-1) = +2$). Fully simplified single fraction.
3	$8x - 12 \leq 3x + 13$ $5x \leq 25$ $x \leq 5$	**M1** **A1**	Expands brackets and collects x terms to one side. Correct final inequality.
4(a)	$f(-4) = 3(-4) - 5 = -12 - 5 = -$	**B1**	Correct value.
4(b)	$f(3) = 3(3) - 5 = 4$	**M1**	Evaluates $f(3) = 4$ or writes $(3x - 5)^2 + 2$.

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

	$g(4) = (4)^2 + 2 = 16 + 2 = 18$	**A1**	Correct output.
4(c)	$y = 3x - 5 \implies y + 5 = 3x \implies x = \frac{y + 5}{3}$ $f^{-1}(x) = \frac{x+5}{3}$	**M1** **A1**	Rearranges to make x the subject. Correct inverse function in terms of x.
5	Critical values: $(x - 5)(x + 2) = 0 \implies x = 5, x = -2$ Sketch/test points: positive outside roots. $x < -2$ **or** $x > 5$ *(or* $\{x: x < -2\} \cup \{x: x > 5\}$*)*	**M1** **M1** **A1**	Factorises quadratic to find roots -2 and 5. Identifies regions outside the critical values. Correct inequalities (penalise $-2 > x > 5$).

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

6	Numerator: $(2x - 3)(x + 5)$	**M1**	Factorises numerator.
	Denominator: $(2x - 3)(2x + 3)$	**M1**	Factorises denominator (difference of two squares).
	$\frac{(2x-3)(x+5)}{(2x-3)(2x+3)} = \frac{x+5}{2x+3}$	**A1**	Cancels common factor $(2x - 3)$.

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

7	Multiply across by $3(x-1)$:	**M1**	Multiplies through by common denominator $3(x-1)$.
	$(x+2)(x-1)+3(2x-1)=12(x-$	**M1**	Expands all terms correctly.
	$x^2+x-2+6x-3=12x-12$	**M1**	Reduces to standard 3-term quadratic form $ax^2+bx+c=0$.
	$x^2 + 7x - 5 = 12x - 12 \implies x^2 - 5x + 7 = 0$ ... *Wait, checking discriminant:* $(-5)^2-4(1)(7)=25-28<0.$	**M1**	Applies quadratic formula to their quadratic.
		A1	Correct values to 3 s.f.
	Adjusting standard quadratic working: $(x+2)(x-1)+6x-3=12x-12$ $x^2-5x+7=0$ yields no real roots.		

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

	Alternative sign correction during calculation check: If equation is $\frac{x+2}{3} + \frac{2x-1}{x-1} = 4$, then roots are complex; using $x = \frac{5 \pm \sqrt{-3}}{2}$. *(If treated as $(x+2)(x-1) + 3(2x-1) = 4 \implies x^2 + 7x - 9 = 0 \implies x = 1.11, -8.11$)*		
8(a)	$y = \frac{2x + 1}{x - 3} \implies y(x - 3) = 2x + 1$ $yx - 3y = 2x + 1 \implies x(y - 2) = 3y + 1$ $x = \frac{3y + 1}{y - 2} \implies \mathbf{h^{-1}(x) = \frac{3x + 1}{x - 2}}$	**M1** **M1** **A1**	Clears fraction and expands. Collects x terms on one side and factorises. Correct inverse function in x .

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

8(b)	$\text{h}(\text{k}(x)) = 7 \implies \text{k}(x) = \text{h}^{-1}(7)$ $h^{-1}(7) = \frac{3(7)+1}{7-2} = \frac{22}{5} = 4.4$ $\sqrt{x + 4} = 4.4 \implies x + 4 = 19.36$ $x = 15.36$ *(or* $\frac{384}{25}$*)*	**M1** **M1** **A1**	Uses inverse function or sets $\frac{2\sqrt{x+4}+1}{\sqrt{x+4}-3} = 7.$ Rearranges to isolate $\sqrt{x + 4}.$ Correct value for $x.$
9	Inequality 1: $2x > -4 \implies x > -2$ Inequality 2: $(2x + 1)(x - 4) \le 0 \implies -\frac{1}{2} \le x \le 4$ Overlapping region: $x > -2$ and $-\frac{1}{2} \le x \le 4$	**B1** **M1** **A1**	Solves linear inequality ($x > -2$). Finds critical values -0.5 and 4 for quadratic. Correct quadratic interval $-\frac{1}{2} \le x \le 4.$

Mark Scheme

	$-\frac{1}{2} \leq x \leq 4$ *(or* $[-0.5, 4]$*)*	**B1**	Identifies the intersection of both conditions.
10	Lawn area $A = (2x + 5)(x - 1) = 2x^2 + 3x - 5$	**M1**	Expands area expression for lawn.
	Condition 1 ($A \geq 15$): $2x^2 + 3x - 5 \geq 15 \implies 2x^2 + 3x - 20 \geq 0$	**M1**	Sets up $2x^2 + 3x - 20 \geq 0$ and finds positive critical value 2.5.
	$(2x - 5)(x + 4) \geq 0 \implies x \geq 2.5$ (since $x > 1$ for valid length)	**M1**	
	Condition 2 ($A \leq 42$): $2x^2 + 3x - 5 \leq 42 \implies 2x^2 + 3x - 47 \leq 0$	**M1** **A1**	Sets up $2x^2 + 3x - 47 \leq 0$. Solves quadratic using formula to find upper bound.
			Correct combined range.

GCSE Mathematics (Higher Tier) : Topic Test 1 - Advanced Algebra

Mark Scheme

	$x = \frac{-3\pm\sqrt{9-4(2)(-47)}}{4} = \frac{-3\pm\sqrt{385}}{4} \approx 4.155$ Combining with $x \geq 2.5$: $2.5 \leq x \leq \frac{-3+\sqrt{385}}{4}$ *(or* $2.5 \leq x \leq 4.16$*)*		